

- 3-2 Consider the three-link planar manipulator shown in Figure 3.23. Derive the forward kinematic equations using the DH convention.

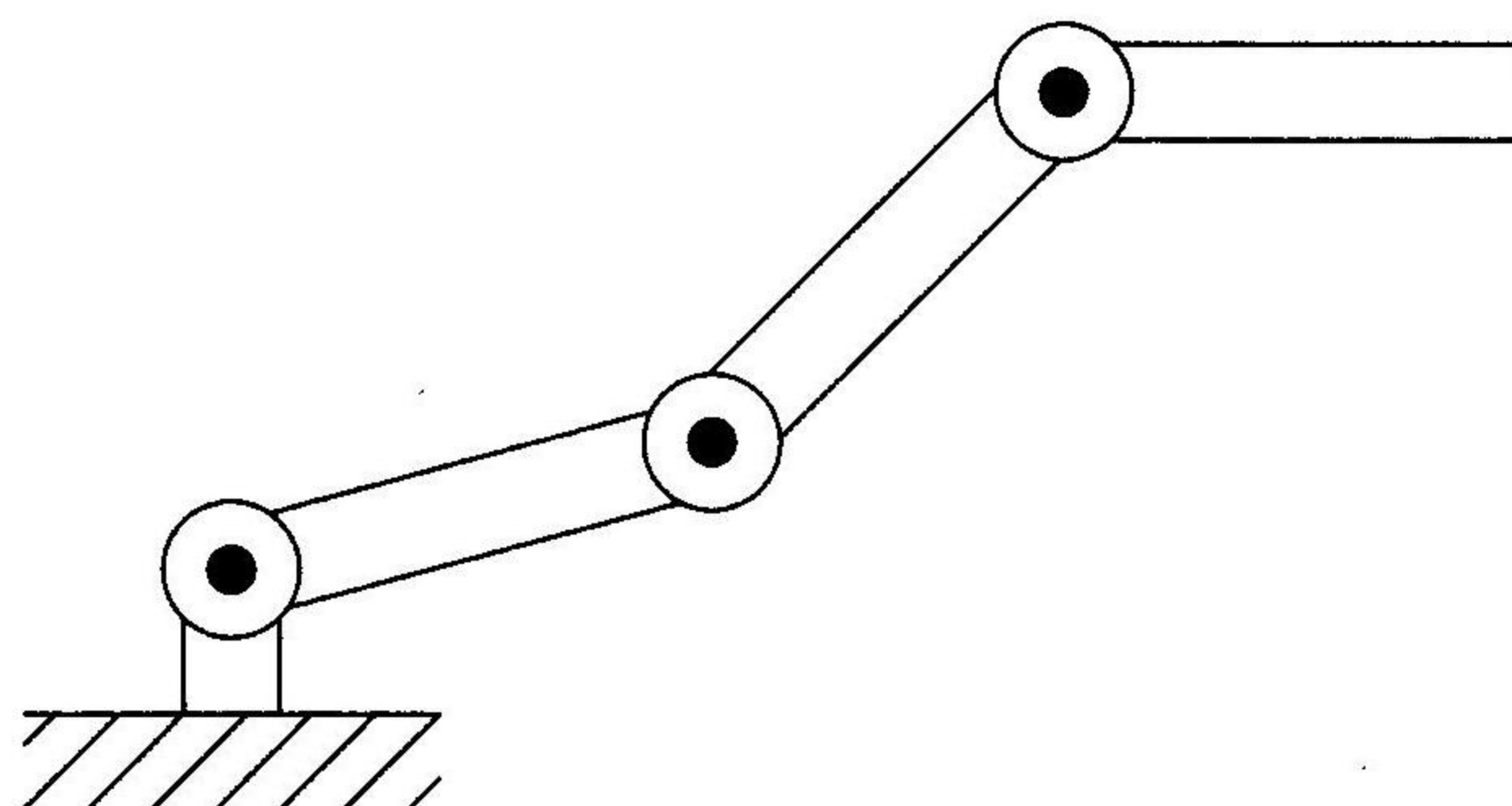


Figure 3.23: Three-link planar arm of Problem 3-2.

- 3-3 Consider the two-link Cartesian manipulator of Figure 3.24. Derive the forward kinematic equations using the DH convention.

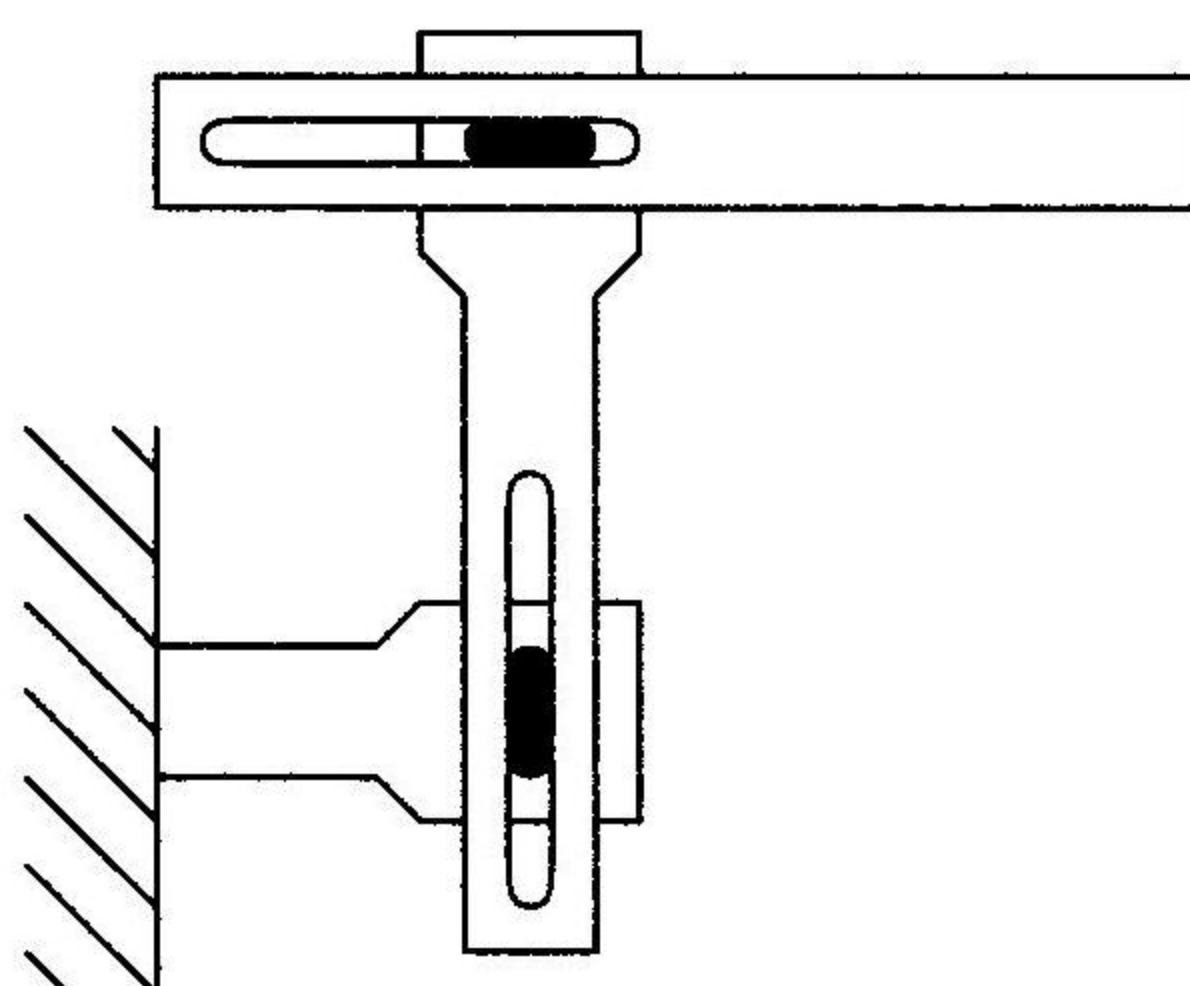


Figure 3.24: Two-link Cartesian robot of Problem 3-3.

- 3-4 Consider the two-link manipulator of Figure 3.25, which has joint 1 revolute and joint 2 prismatic. Derive the forward kinematic equations using the DH convention.

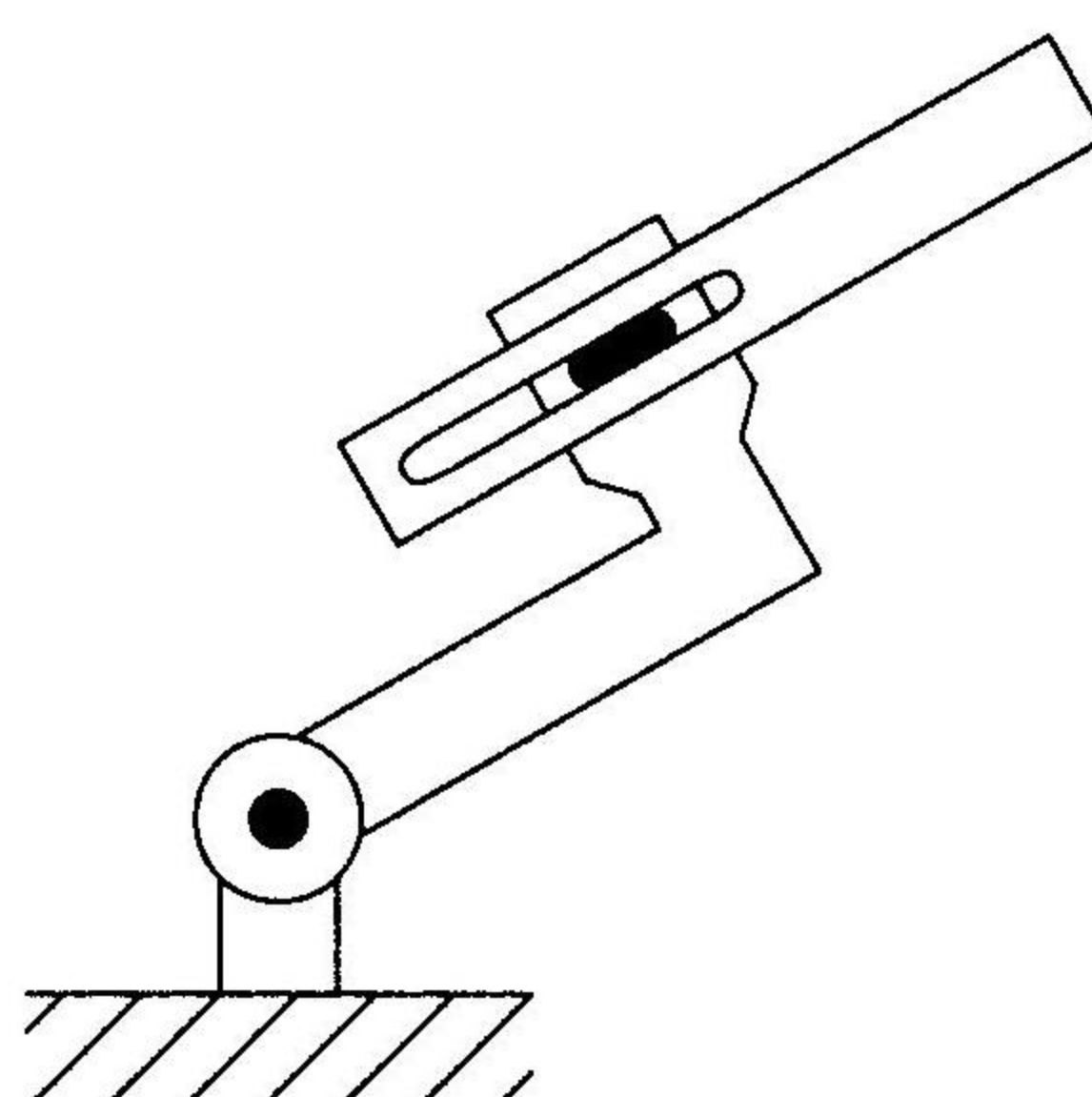


Figure 3.25: Two-link planar arm of Problem 3-4.

3-5 Consider the three-link planar manipulator of Figure 3.26. Derive the forward kinematic equations using the DH convention.

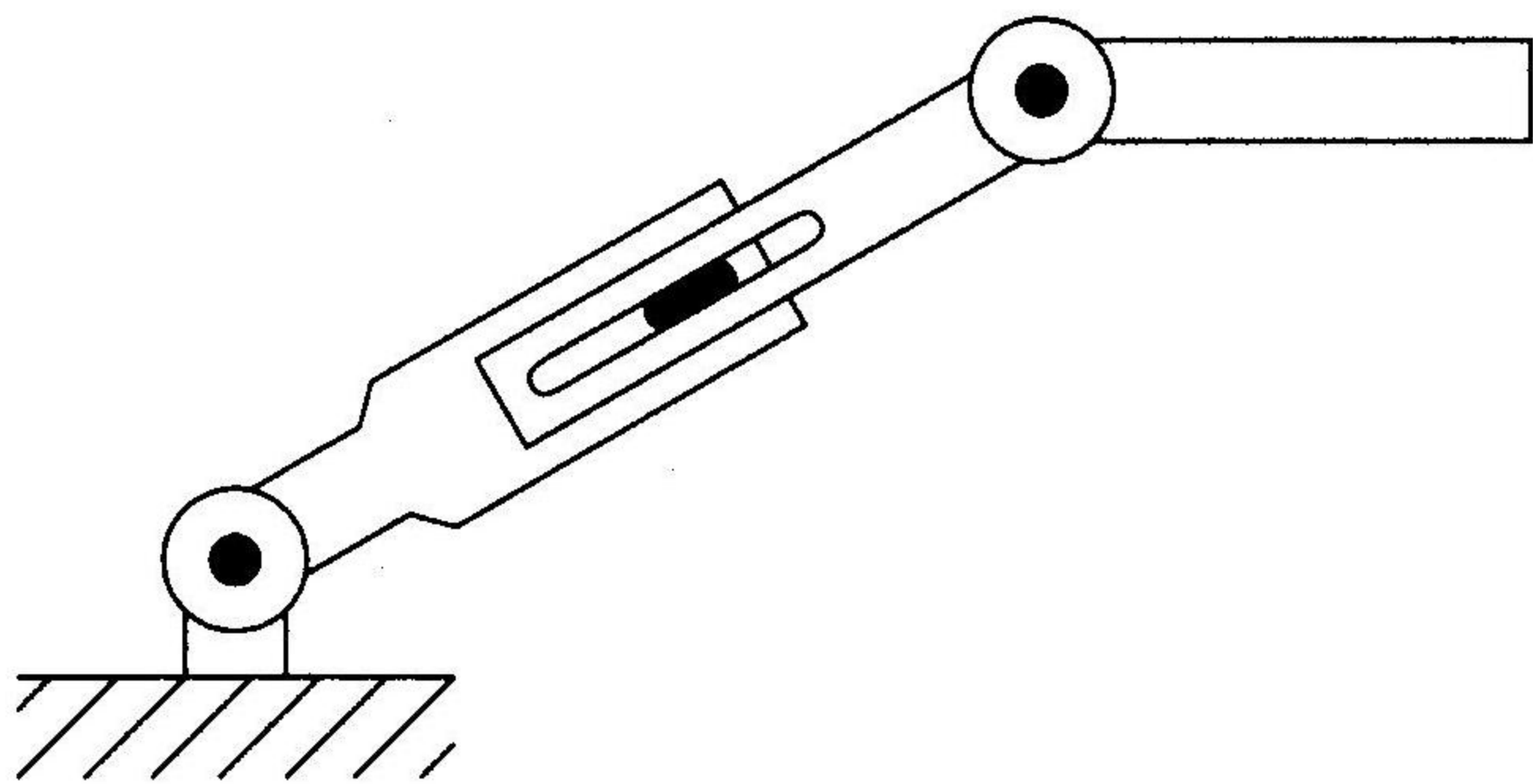


Figure 3.26: Three-link planar arm with prismatic joint of Problem 3-5.

3-6 Consider the three-link articulated robot of Figure 3.27. Derive the forward kinematic equations using the DH convention.

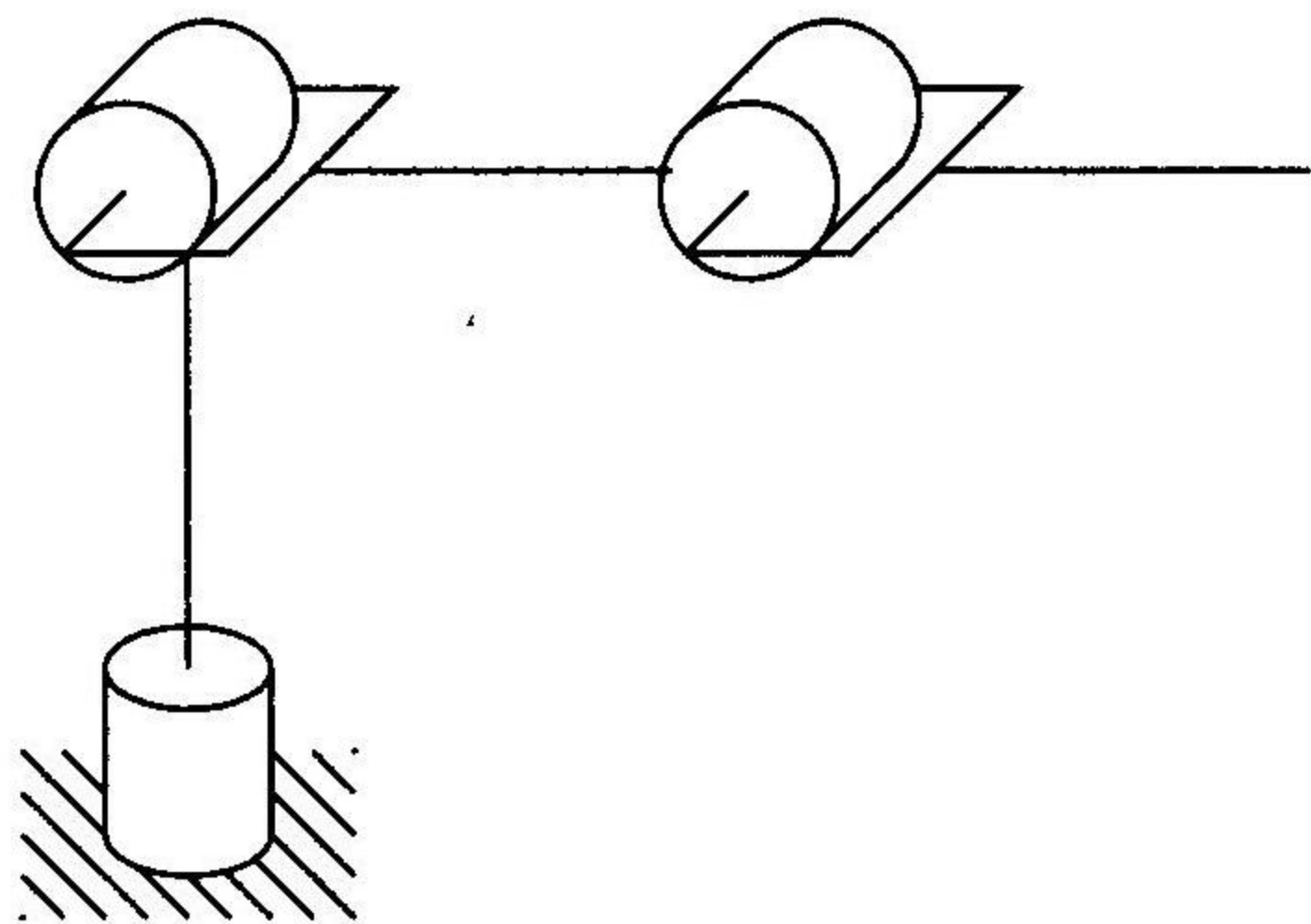


Figure 3.27: Three-link articulated robot.

3-7 Consider the three-link Cartesian manipulator of Figure 3.28. Derive the forward kinematic equations using the DH convention.

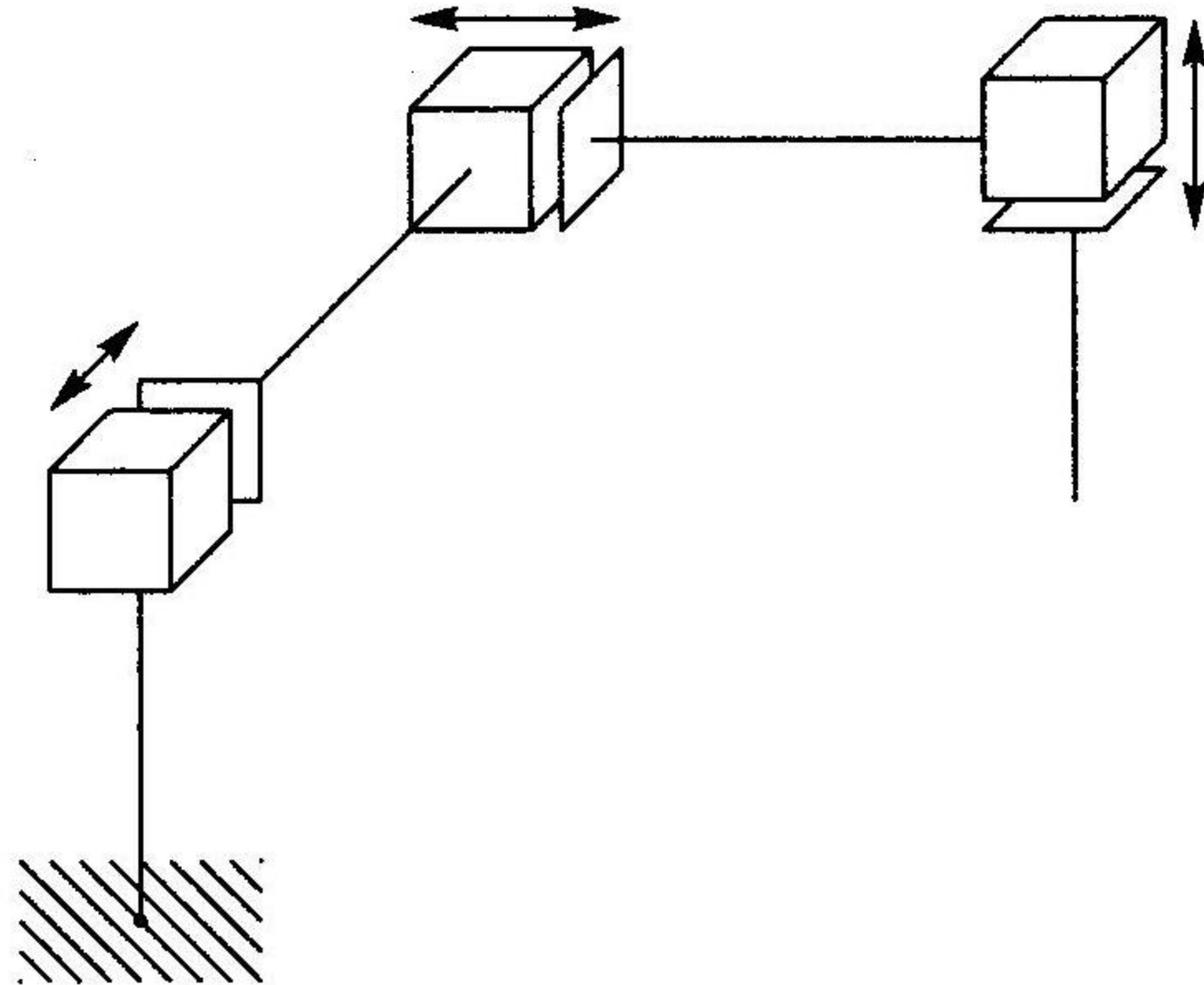


Figure 3.28: Three-link Cartesian robot.



- 3-8 Attach a spherical wrist to the three-link articulated manipulator of Problem 3-6 as shown in Figure 3.29. Derive the forward kinematic equations for this manipulator.

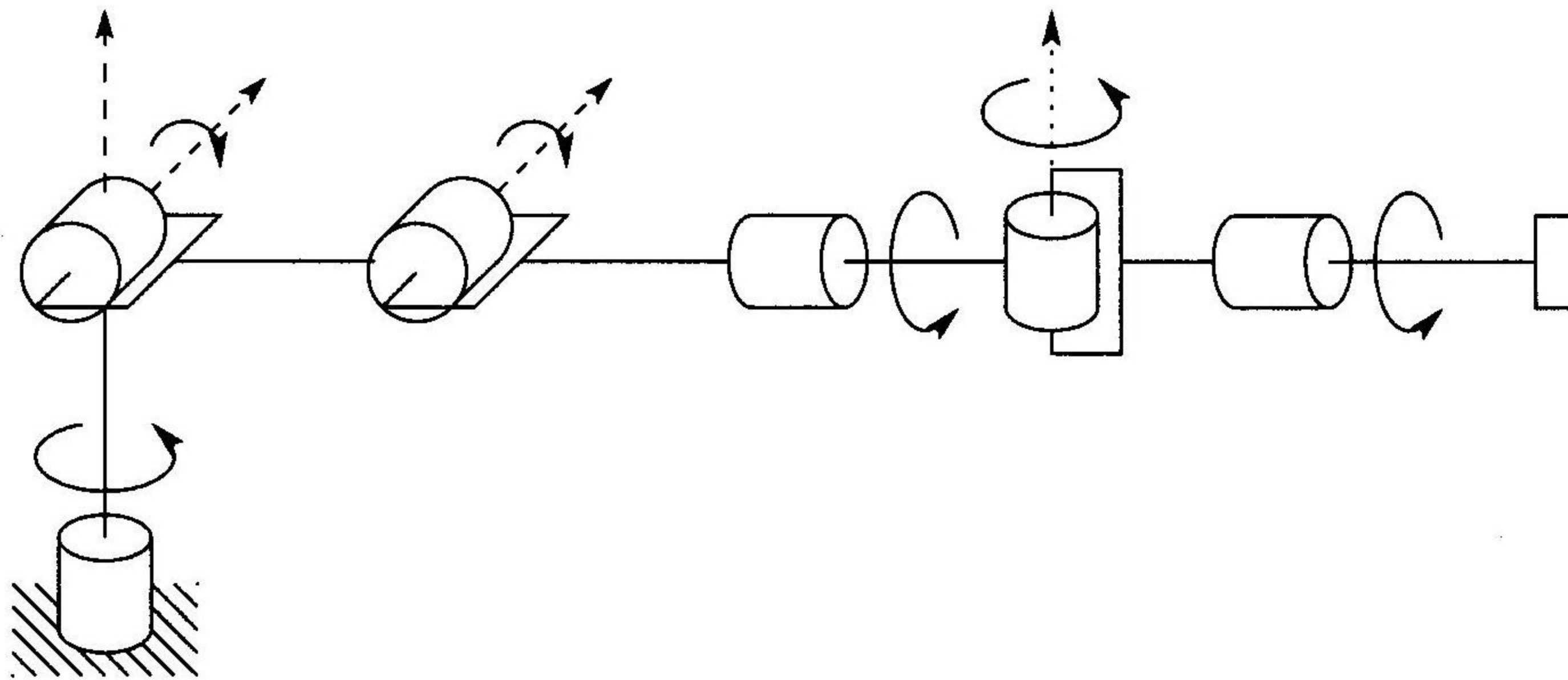


Figure 3.29: Elbow manipulator with spherical wrist.

- 3-9 Attach a spherical wrist to the three-link Cartesian manipulator of Problem 3-7 as shown in Figure 3.30. Derive the forward kinematic equations for this manipulator.

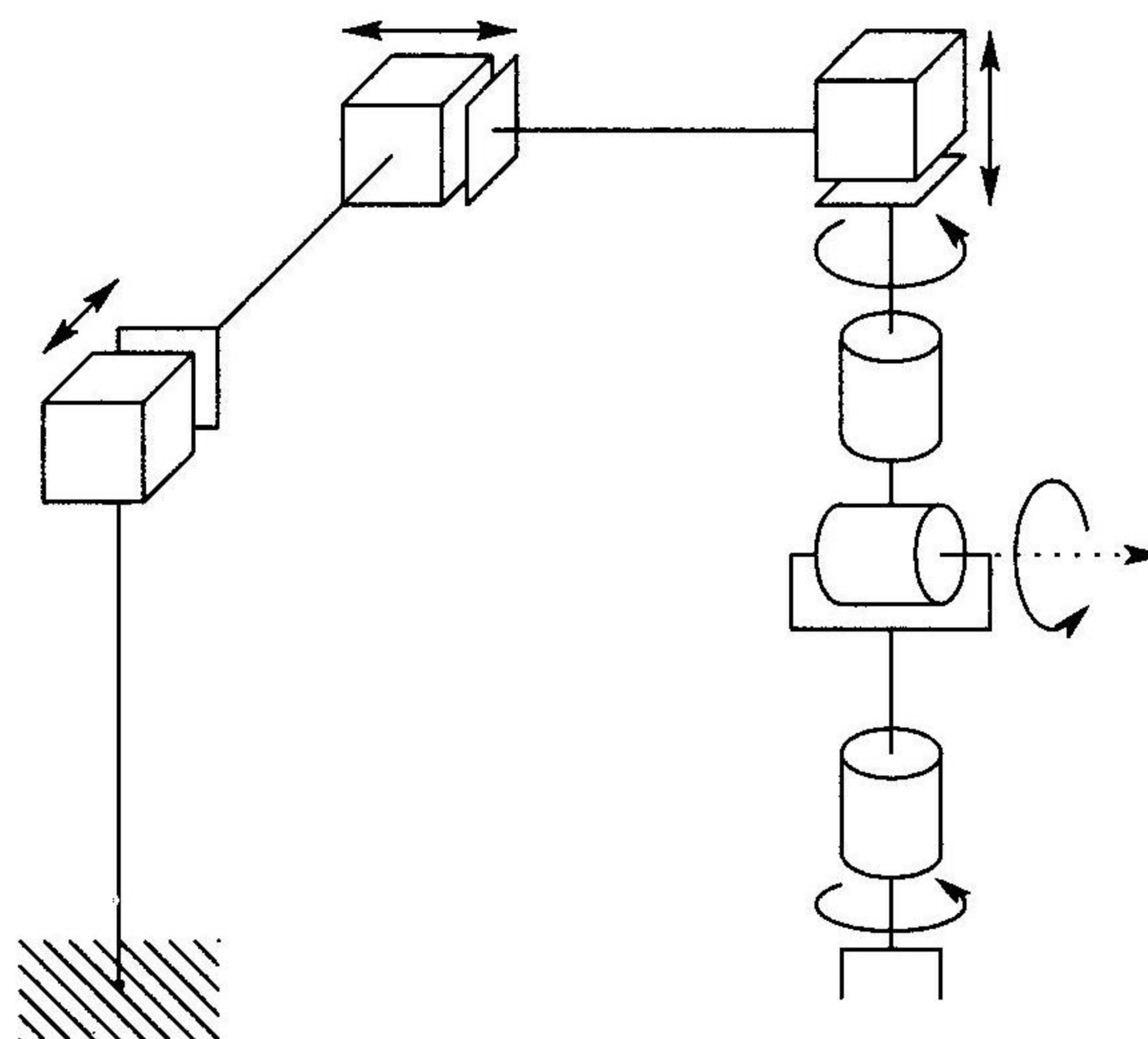


Figure 3.30: Cartesian manipulator with spherical wrist.

- 3-10 Consider the PUMA 260 manipulator shown in Figure 3.31. Derive the complete set of forward kinematic equations by establishing appropriate DH coordinate frames, constructing a table of DH parameters, forming the A matrices, etc.

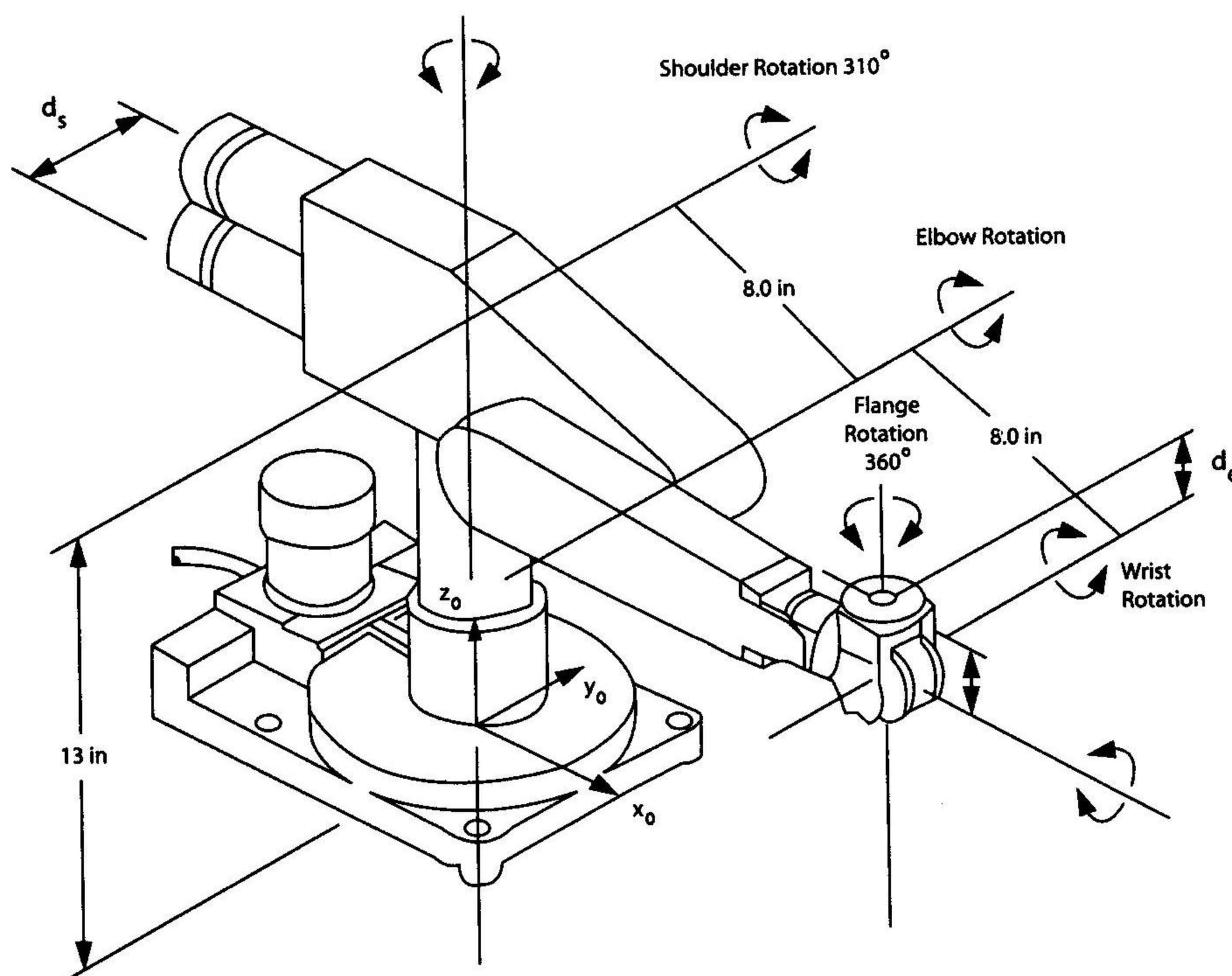


Figure 3.31: PUMA 260 manipulator.

- 3-11 Given a desired position of the end effector, how many solutions are there to the inverse kinematics of the three-link planar arm shown in Figure 3.32? If the orientation of the end effector is also specified, how many solutions are there? Use the geometric approach to find them.

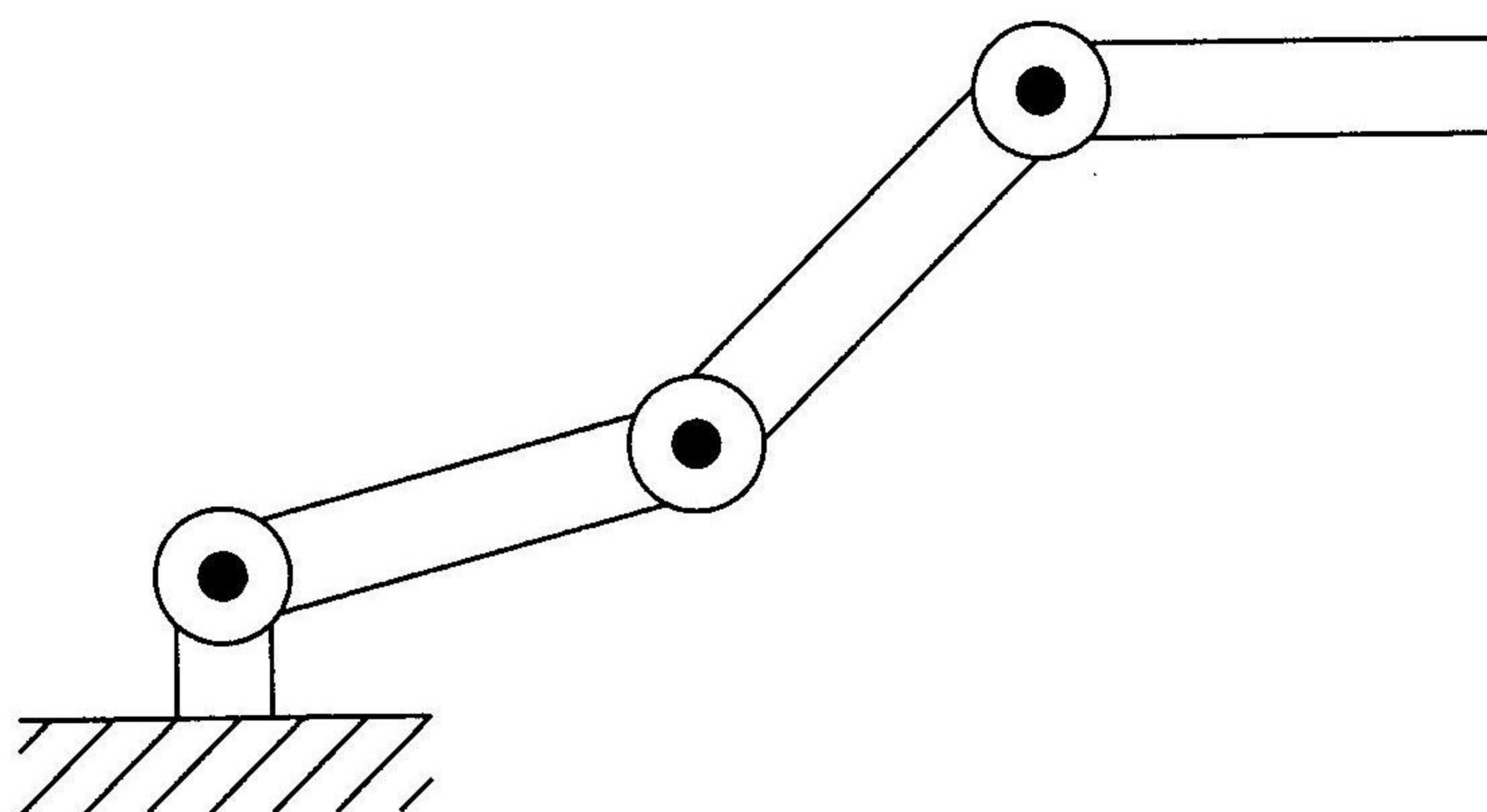


Figure 3.32: Three-link planar robot with revolute joints.



3-12 Repeat Problem 3-11 for the three-link planar arm with prismatic joint of Figure 3.33.

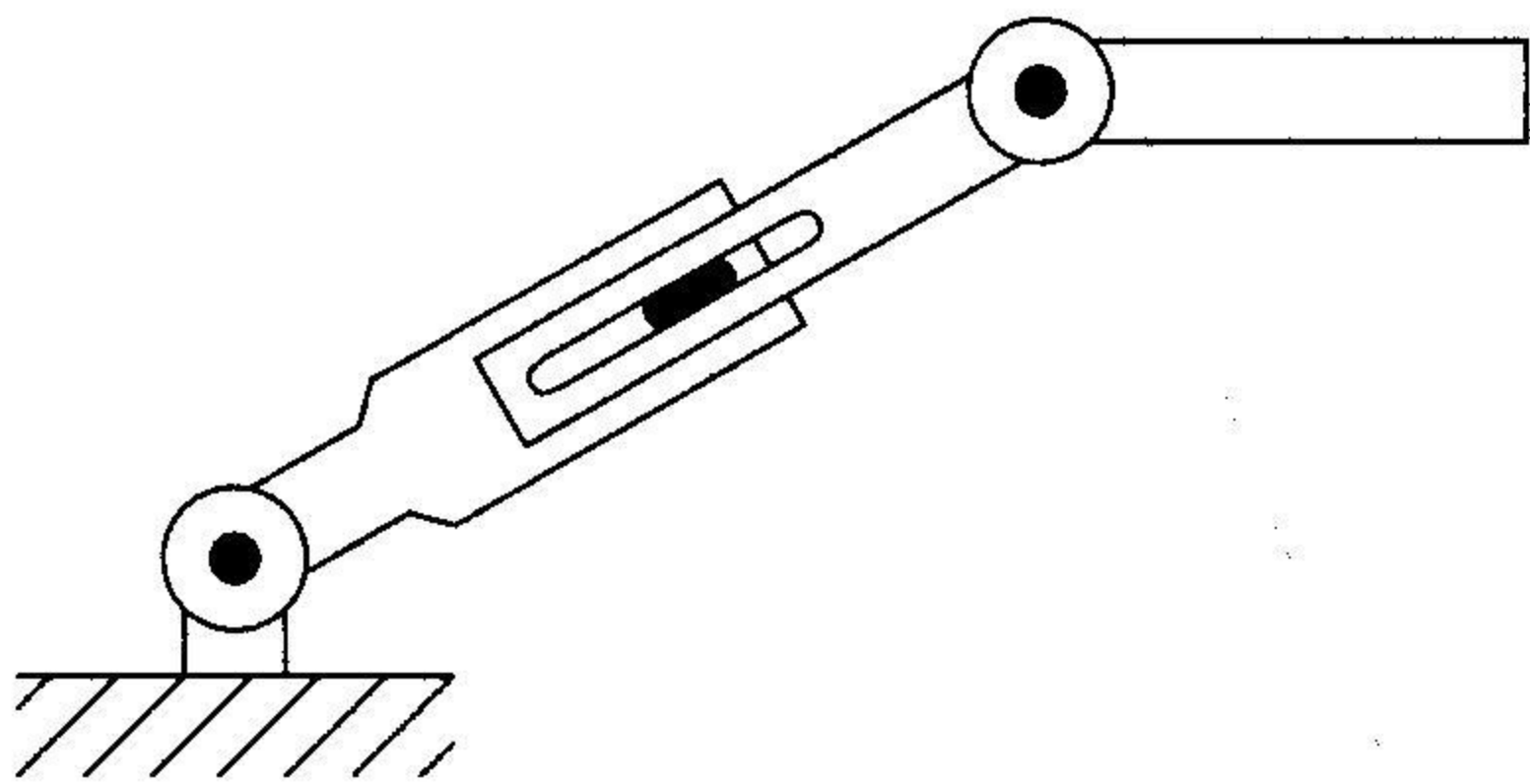


Figure 3.33: Three-link planar robot with prismatic joint.

3-13 Solve the inverse position kinematics for the cylindrical manipulator of Figure 3.34.

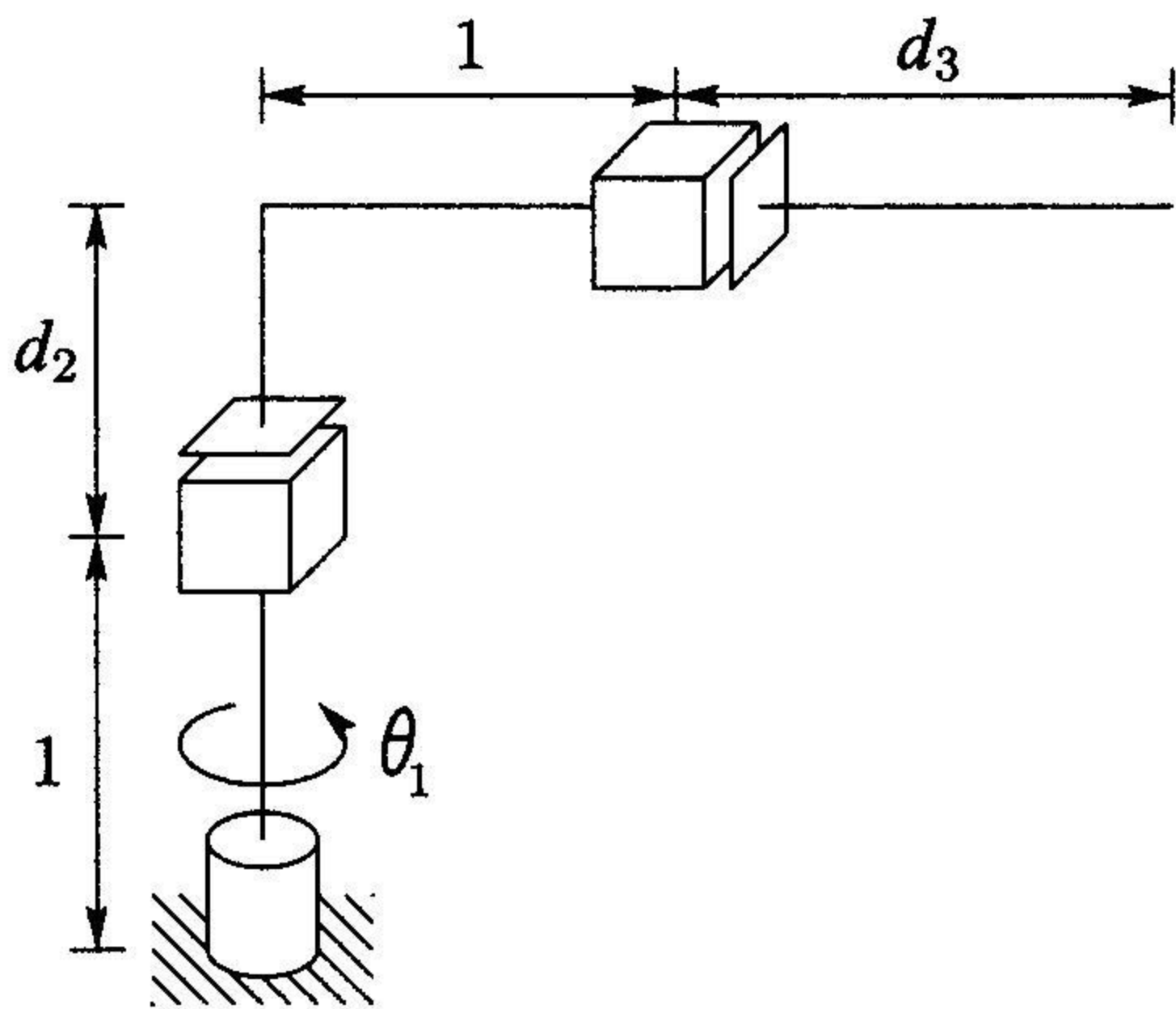


Figure 3.34: Cylindrical configuration.

3-14 Solve the inverse position kinematics for the Cartesian manipulator of Figure 3.35.

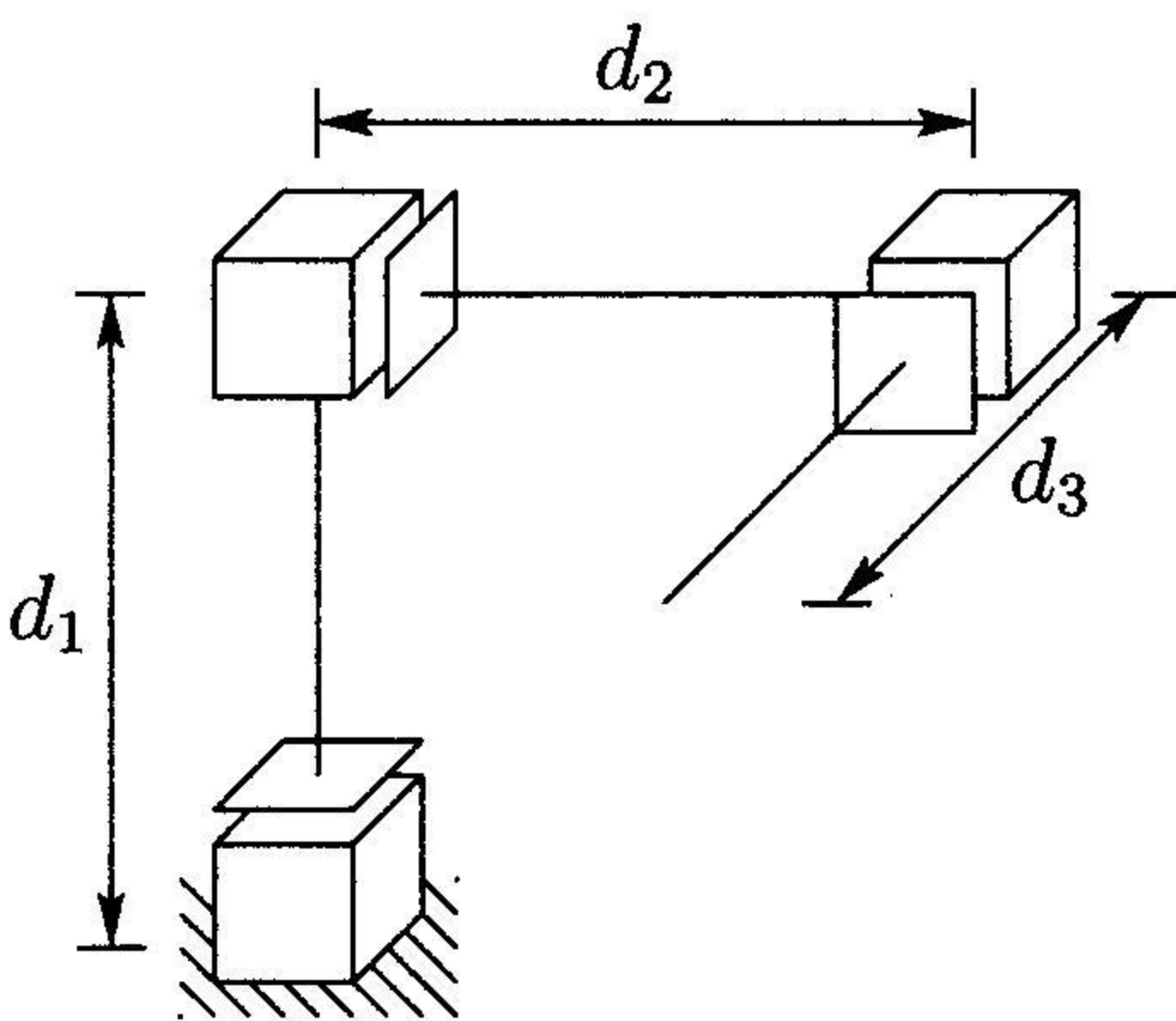


Figure 3.35: Cartesian configuration.



- 3-15 Add a spherical wrist to the three-link cylindrical arm of Problem 3-13 and write the complete inverse kinematics solution.
- 3-16 Add a spherical wrist to the Cartesian manipulator of Problem 3-14 and write the complete inverse kinematics solution.
- 3-17 Write a computer program to compute the inverse kinematic equations for the elbow manipulator using Equations (3.64)–(3.69). Include procedures for identifying singular configurations and choosing a particular solution when the configuration is not singular. Test your routine for various special cases, including singular configurations.
- 3-18 The Stanford manipulator of Example 3.5 has a spherical wrist. Given a desired position  $o$  and orientation  $R$  of the end effector,
1. Compute the desired coordinates of the wrist center  $o_c^0$ .
  2. Solve the inverse position kinematics, that is, find values of the first three joint variables that will place the wrist center at  $o_c$ . Is the solution unique? How many solutions did you find?
  3. Compute the rotation matrix  $R_3^0$ . Solve the inverse orientation problem for this manipulator by finding a set of Euler angles corresponding to  $R_6^3$  given by Equation (3.52).
- 3-19 Repeat Problem 3-18 for the PUMA 260 manipulator of Problem 3-10, which also has a spherical wrist. How many total solutions did you find?
- 3-20 Find all other solutions to the inverse kinematics of the elbow manipulator of Example 3.9.
- 3-21 Modify the solutions  $\theta_1$  and  $\theta_2$  for the spherical manipulator given by Equations (3.47) and (3.49) for the case of a shoulder offset.

## NOTES AND REFERENCES

The Denavit-Hartenberg convention for assigning coordinate frames was introduced in the fifties, and is described in [57] and [27]. Since then, many articles have been written on the topics of forward and inverse kinematics. Seminal articles that deal with forward kinematics include [19], [29], [74], [75], [103], [57], and [138]. Inverse kinematics problems are considered in [6], [45], [53], [75], [76], [103], [105], [113], and [134]. In the late seventies and